Closing Wed: HW_6A,6B,6C (7.4,7.5,7.7)
Office Hours Today: 1:30-3:00pm (Smith 309)

Entry Task: Evaluate

1. $\int \frac{x^2+1}{x^2-2x-3}\,dx$

2. $\int \frac{x}{x^2+4x+5}\,dx$

How to integrate

A. Look for simplifications/substitutions

B. Products/Logs/Inverse Trig → BY PARTS
   Sin/Cos/Tan/Sec combos → TRIG
   Quadratic (under a radical) → TRIG SUB
   Rational Function → PART. FRAC.

C. If nothing seems to work, **substitution**.
   (u = inside, u = $\sqrt{\ \ }$, u = trig, u = e^x)

Examples of substitution:

1. $\int e^{\sqrt{x}}dx$

2. $\int \frac{3}{x-2\sqrt{x}}dx$

3. $\int \frac{\cos(x)}{4-\sin^2(x)}dx$

4. $\int e^x \cos(\mathrm{e}^{\mathrm{x}}) \sin^3(e^x)\, dx$

How would you *start* these?

1. $\int \tan^3(x)\sec(x)\,dx$

2. $\int x^2 \ln(x)\,dx$

3 $\int x\sqrt{5 - x^2}dx$

4. $\int \frac{\sqrt{x^2 - 1}}{x^2}dx$

5. $\int \frac{x^2 + 1}{x^2 - 2x - 3}dx$

6. $\int x\tan^{-1}(x)dx$

7. $\int \frac{dx}{\sqrt{4x^2 + 8x - 12}}dx$

7.7 Approximating Integrals:

Despite our best efforts in 7.1-7.5**,** the vast majority of integrals CANNOT be done with any of our methods.

So we usually have to approximate!

In this section we add two more approximation methods that are slightly more accurate. We already know left, right, and midpoint methods (but I included them below for completeness).

To approximate $\int_a^b f(x)dx$

1. Pick **n = number of subdivisions**.

   Compute $\Delta x = \frac{b-a}{n}$.
2. Label the tick marks: $x_i = a + i\Delta x$
3. Use an approximation method:

$$L_n = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_{n-1})] \quad \text{(Left endpoint)}$$
$$R_n = \Delta x[f(x_1) + f(x_2) + \cdots + f(x_n)] \quad \text{(Right endpoint)}$$
$$M_n = \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)] \quad \text{(Midpoint)}$$

New - Trapezoid Rule: (all the "middle terms" are multiplied by 2)

$$T_n = \frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

New - Simpson's Rule: *n* must be even! (Alternating multiplying middle terms by 4 and 2)

$$S_n = \frac{1}{3}\Delta x[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Example: (Note: None of our methods can integrate this)

Estimate

$$\int_0^3 \sqrt{100 - x^3}dx$$

Here are how to use the left, right, midpoint and trapezoid rules with n = 3 subdivisions:

$$L_3 = (1)\left[\sqrt{100-(0)^3} + \sqrt{100-(1)^3} + \sqrt{100-(2)^3}\right] \approx 29.5415$$

$$R_3 = (1)\left[\sqrt{100-(1)^3} + \sqrt{100-(2)^3} + \sqrt{100-(3)^3}\right] \approx 28.0855$$

$$M_3 = (1)\left[\sqrt{100-(0.5)^3} + \sqrt{100-(1.5)^3} + \sqrt{100-(2.5)^3}\right] \approx 29.0091$$

NEW – Trapezoid rule with n = 3.

$$T_3 = \frac{1}{2}(1)\left[\sqrt{100-(0)^3} + 2\sqrt{100-(1)^3} + 2\sqrt{100-(2)^3} + \sqrt{100-(3)^3}\right] \approx 28.8135$$

NEW – Simpson's rule with n = 6 (n must be even)

$$S_6 = \frac{1}{3}\cdot\frac{1}{2}\Big[\sqrt{100-(0)^3} + 4\sqrt{100-(0.5)^3} + 2\sqrt{100-(1)^3} + 4\sqrt{100-(1.5)^3}$$
$$+2\sqrt{100-(2)^3} + 4\sqrt{100-(2.5)^3} + \sqrt{100-(3)^3}\Big] \approx 28.9441$$

"Actual" Value (to 8 places after the decimal): 28.94418784

Example:
With n = 4, use both new methods to approximate (just set up)

$$\frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-\frac{1}{2}x^2}\,dx$$

$\Delta \boldsymbol{x} =$ $\quad$ $, \boldsymbol{x_0} =$ $\quad$ $, \boldsymbol{x_1} =$ $\quad$,
$\boldsymbol{x_2} =$ $\quad$ $, \boldsymbol{x_3} =$ $\quad$ $, \boldsymbol{x_4} =$ $\quad$.

$$\frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)]$$

$=$

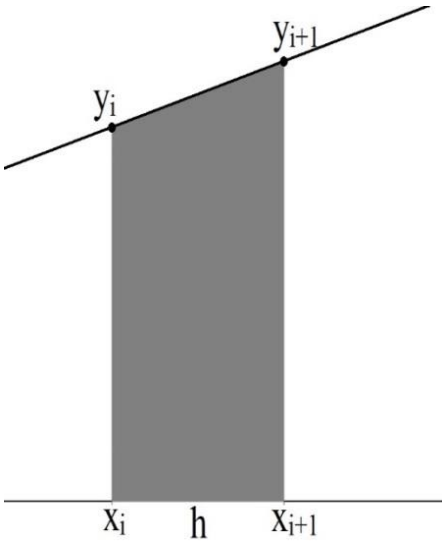
$$\frac{1}{3}\Delta x[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

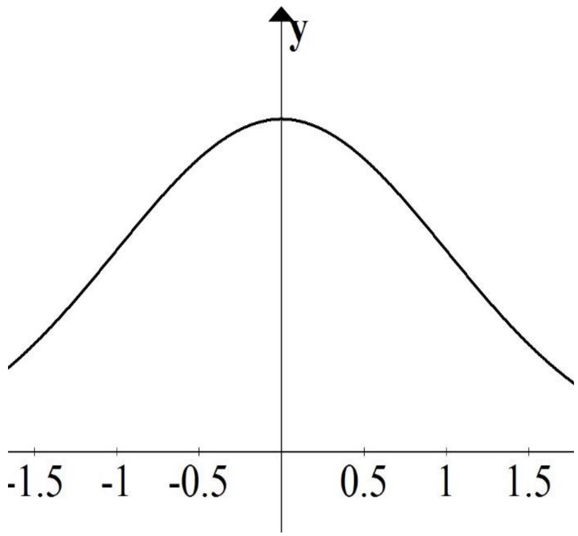
$=$

7.7 Quick Derivation Notes

Trapezoid Rule:

$$\text{Shaded Area} = \frac{h}{2}(y_i + y_{i+1})$$





Simpson's Rule: If the curve below is a ***parabola*** (y = ax^2 + bx + c) that goes through the three indicated points, then

$$\text{Shaded Area} = \frac{h}{3}(y_i + 4y_{i+1} + y_{i+2})$$